V. CONCLUSIONS

A study has been made on the EGW circulator, which has led to the fabrication of a broad-band MIC circulator operating in the frequency range 8–12 GHz. The principal scope of the study was to establish the fundamental physical principles which underlie the EGC's operation and to show how they differ from those relative to the traditional Y-junction MIC circulators. It has been shown that broad-band operation of the EGC requires the introduction of mode-suppressing devices. Our choice was an inhomogeneous magnetic bias. Although the performance data of a three-port EGC favorably compare to those of other circulators, a greater complexity in mechanical construction as well as higher and more extended magnetic biases are necessary to guarantee a correct operation of the EGC. These are perhaps the principal reasons why such circulators as the continuous tracking circulator (CTC) will more likely enjoy the favor of microwave designers. For broad-band four-port circulators, the EGC version seems to be much more attractive mainly because no continuous tracking principle has been demonstrated to exist for this case [10]. The optimization of a four-port EGC is, however, still under way. The theory of the EGC seems to be at a rather early stage and still deserves some attention.

ACKNOWLEDGMENT

The authors wish to thank Dr. C. Misiano for the construction of the circulators.

REFERENCES

[1] M. E. Hines, "Ferrite phase shifters and multiport circulators in microstrip and strip line," in *Dig. of Tech. Papers, 1971 IEEE G-MTT Int. Microwave Symp.* (Washington, D. C.), pp. 108–109.

[2] ——, "Reciprocal and nonreciprocal modes of propagation in ferrite stripline and microstrip devices," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-19, pp. 442–451, May 1971.

[3] ——, "Ferrite transmission devices using the edge-guided modes of propagation," in *Dig. of Tech. Papers, 1972 IEEE G-MTT Int. Microwave Symp.* (Chicago, Ill.), pp. 236–237.

[4] P. de Santis and F. Pucci, "Novel type of M. I. C. symmetrical three-port circulator," *Electron. Lett.*, vol. 8, pp. 12–13, Jan. 1972.

[5] ——, "Experiments on the optimization of a novel M. I. C. symmetrical three-port circulator," in *Dig. of Tech. Papers, 1972 IEEE G-MTT Int. Microwave Symp.* (Chicago, Ill.), pp. 238–240.

[6] M. Blanc, L. Dusson, and J. Guidevaux, "Etudes de dispositifs non reciproques a ferrite a trés large bande. Premieres realizations," *Rev. Tech. Thomson-CSF*, vol. 4, pp. 27–48, Mar. 1972.

[7] Y. S. Wu and F. J. Rosenbaum, "Wideband operation of microstrip circulators," in *Dig. Tech. Papers, IEEE G-MTT 1973 Int. Microwave Symp.* (Boulder, Colo.), pp. 92–94.

[8] P. de Santis, "Edge guided modes in ferrite microstrips with curved edges," *Appl. Phys.*, vol. 4, pp. 167–174, Aug. 1974.

[9] D. Massé, "Broadband microstrip junction circulators," *Proc. IEEE* (Lett.), vol. 56, pp. 352–353, Mar. 1968.

[10] W. H. Ku and Y. S. Wu, "On stripline four port circulator," in *Dig. of Tech. Papers, IEEE G-MTT 1973 Int. Microwave Symp.* (Boulder, Colo.), pp. 86–88.

Measurements of Intercavity Couplings

A. E. ATIA, MEMBER, IEEE, AND
A. E. WILLIAMS, MEMBER, IEEE

***Abstract*—This short paper describes the determination of couplings within a system of coupled cavities by measuring frequencies at which the phase of the input reflection coefficient is either 0° or 180°. A high degree of accuracy may be achieved and corrections can be made for finite cavity Q.**

Manuscript received September 19, 1974; revised December 23, 1974. This short paper is based upon work performed in COMSAT Laboratories under the sponsorship of the International Telecommunications Satellite Organization (INTELSAT). Views expressed in this short paper are not necessarily those of INTELSAT.

The authors are with the COMSAT Laboratories, Clarksburg, Md. 20734.

INTRODUCTION

Accurate determination of coupling between electrical cavities is required to design direct-coupled cavity waveguide bandpass filters. Bethe's [1] small-hole coupling theory, modified for larger slots by Cohn [2], gives dimensions which are approximately correct and normally sufficient for filter transfer functions having a small number of cavities and monotic out-of-band responses.

However, for bandpass filters which have real transmission zeros, more stringent specifications are imposed on the accuracy of the coupling values. Therefore, to realize this improved accuracy, it is desirable to have some method of measuring the coupling values within the assembled filter. This short paper describes the use of measurements of the input reflection coefficient phase from a short-circuited set of cavities to determine intercavity coupling. The accuracy is sufficient for successful tuning of such filters as the non-minimum-phase optimum-amplitude bandpass waveguide filter [3].

MEASUREMENT TECHNIQUE

A general lumped-element equivalent circuit for a system of n coupled cavities is shown in Fig. 1. All resonant cavities are tuned to a resonant frequency $\omega_0 = 1/(LC)^{1/2} = 1$ rad/s and have the same impedance, $Z_0 = (L/C)^{1/2} = 1\ \Omega$. Use of a narrow-band approximation makes it possible to describe the coupling between the cavities as an $n \times n$, symmetric, purely imaginary matrix jM, which is frequency independent near ω_0. The element M_{ij} is the coupling between the ith and jth cavities.

The input impedance $Z_{11}^{(n)}$ is given by

$$Z_{11}^{(n)} = \frac{\det (j\lambda I - jM_n)}{\det (j\lambda I - jM_{n-1})} \tag{1}$$

where M_{n-1} is the matrix resulting from the deletion of the first row and column of M_n,; $\lambda = s + (1/s)$, $s = j\omega$, and I is the identity matrix. Therefore, the poles and zeros of $Z_{11}^{(n)}$ are the eigenvalues of M_{n-1} and M_n, respectively.

In practice the reflection coefficient $\rho^{(n)}$, which is equal to $[Z_{11}^{(n)} - R]/[Z_{11}^{(n)} + R]$, is the parameter which is most easily measured. It follows that the 0° and 180° phase positions of $\rho^{(n)}$ correspond exactly to the poles and zeros of $Z_{11}^{(n)}$. The measuring technique used exploits the fact that frequencies of the zeros and poles of the input impedance can be measured with a high degree of accuracy. Then the coupling values are computed from a knowledge of these frequencies and the way in which the cavities are coupled. Analytically, this problem is equivalent to determining the values of the elements of a real symmetric matrix M from a knowledge of its eigenvalues and the eigenvalues of one of its first-order minors. Explicit expressions for the coupling coefficients are presented for some cases of practical importance (i.e., $n = 2$, 3, and 4). Additionally, a method for computing the couplings in the general case of arbitrary n is also described.

Two Cavities ($n = 2$)

The condition for synchronous tuning of the cavities is

$$\omega_p - \omega_{z1} = \omega_{z2} - \omega_p \tag{2}$$

where $\omega_{z1,2}$ and ω_p are the angular frequencies of the zeros and pole of $Z_{11}^{(2)}$, respectively. Under this condition, the input impedance is given by

$$Z_{11}^{(2)} = j\,\frac{\lambda^2 - M_{12}^2}{\lambda}. \tag{3}$$

The zeros are at $\lambda_z = \pm M_{12}$, and the poles are at $\lambda_p = 0$.

It can be seen from this equation that coupling M_{12} can be computed by accurately measuring the zeros of $Z_{11}^{(2)}$. Fig. 2(a) is a typical polar plot of the locus of $\rho^{(2)}$ as a function of frequency. Since

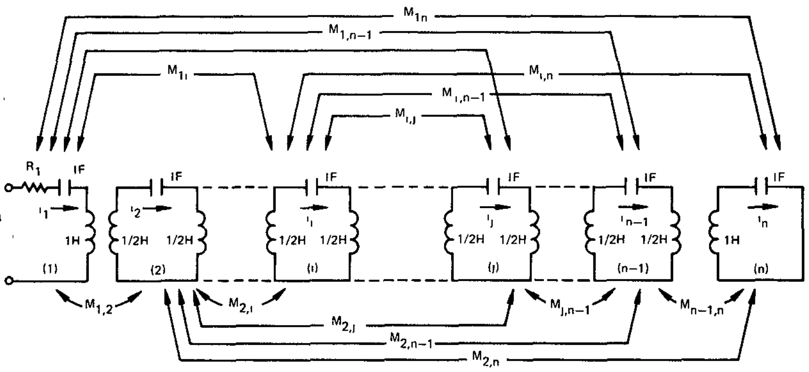

Fig. 1. General equivalent circuit of n arbitrarily coupled cavities.

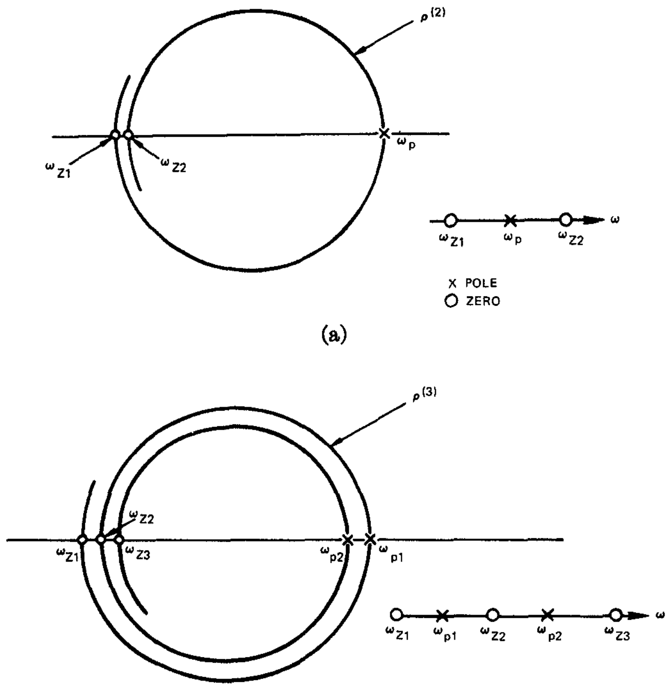

Fig. 2. (a) Typical polar plot of the locus of $\rho^{(2)}$ as a function of frequency. (b) Typical polar plot of the locus of $\rho^{(3)}$ as a function of frequency.

$$\lambda = \left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) \cong \frac{2}{\omega_0}(\omega - \omega_0) \tag{4}$$

then

$$\lambda_{z1} - \lambda_{z2} = 2M_{12} = \frac{2(\omega_{z1} - \omega_{z2})}{\omega_0}$$

or

$$M_{12} = \frac{(\omega_{z1} - \omega_{z2})}{\omega_0}. \tag{5}$$

Therefore the coupling M_{12} between cavities 1 and 2 can be determined by measuring ω_{z1} and ω_{z2} with $\omega_p = \omega_0$.

Three Cavities ($n = 3$)

The conditions for synchronous tuning are

$$\omega_{z2} = \omega_0 \tag{6a}$$

$$\omega_{z2} - \omega_{z1} = \omega_{z3} - \omega_{z2} \tag{6b}$$

$$\omega_{z2} - \omega_{p1} = \omega_{p2} - \omega_{z2}. \tag{6c}$$

The input impedance (with $M_{13} = 0$) is

$$Z_{11}^{(3)} = j\,\frac{\lambda^3 - \lambda(M_{23}^2 + M_{12}^2)}{\lambda^2 - M_{23}^2} \tag{7}$$

with zeros $\lambda_z = 0, \pm(M_{23}^2 + M_{12}^2)^{1/2}$, and poles $\lambda_p = \pm M_{23}$. Fig. 2(b) is a plot of $\rho^{(3)}$ as a function of frequency.

The coupling values M_{23} and M_{12} can be computed by using (4) as follows:

$$M_{23} = \frac{(\omega_{p1} - \omega_{p2})}{\omega_0} \tag{8}$$

and

$$M_{12} = \frac{(\Delta\omega_{Z13}^2 - \Delta\omega_{p12}^2)^{1/2}}{\omega_0} \tag{9}$$

where ω_{z2} is equal to ω_0 and $\Delta\omega_{z13} = \omega_{z1} - \omega_{z3}$, $\Delta\omega_{p12} = \omega_{p1} - \omega_{p2}$.

Four Cavities ($n = 4$)

For four cavities, a unique solution is possible if only three couplings are present. In this case, the conditions for synchronous tuning of the cavities are

$$\omega_0 = \omega_{p2} \tag{10a}$$

$$\omega_{p2} - \omega_{z1} = \omega_{z4} - \omega_{p2} \tag{10b}$$

$$\omega_{p2} - \omega_{p1} = \omega_{p3} - \omega_{p2} \tag{10c}$$

$$\omega_{p2} - \omega_{z2} = \omega_{z3} - \omega_{p2}. \tag{10d}$$

The solution for the coupling coefficients in this case yields the following results:

Case i: $M_{12} = M_{13} = M_{24} = 0$

$$M_{14}^2 = \left(\frac{\omega_{z4} - \omega_0}{\omega_0}\right)^2 - \left(\frac{\omega_{z3} - \omega_0}{\omega_0}\right)^2 - \left(\frac{\omega_{p3} - \omega_0}{\omega_0}\right)^2$$

$$M_{23} = \frac{[(\omega_{z3} - \omega_0)/\omega_0][(\omega_{z4} - \omega_0)/\omega_0]}{M_{14}}$$

$$M_{34}^2 = \left(\frac{\omega_{p3} - \omega_0}{\omega_0}\right)^2 - M_{23}^2.$$

Case ii: $M_{23} = M_{13} = M_{24} = 0$

$$M_{34} = \frac{\omega_{p3} - \omega_0}{\omega_0}$$

$$M_{12} = \frac{[(\omega_{z3} - \omega_0)/\omega_0][(\omega_{z4} - \omega_0)/\omega_0]}{M_{34}}$$

$$M_{14}^2 = \left(\frac{\omega_{z4} - \omega_0}{\omega_0}\right)^2 - \left(\frac{\omega_{z3} - \omega_0}{\omega_0}\right)^2 - M_{34}^2 - M_{12}^2.$$

Case iii: $M_{34} = M_{12} = M_{24} = 0$

$$M_{23} = \frac{\omega_{p3} - \omega_0}{\omega_0}$$

$$M_{14} = \frac{[(\omega_{z3} - \omega_0)/\omega_0][(\omega_{z4} - \omega_0)/\omega_0]}{M_{23}}$$

$$M_{12}^2 = \left(\frac{\omega_{z4} - \omega_0}{\omega_0}\right)^2 - \left(\frac{\omega_{z3} - \omega_0}{\omega_0}\right)^2 - M_{23}^2 - M_{14}^2.$$

Case iv: $M_{14} = M_{13} = M_{24} = 0$

$$M_{12}^2 = \left(\frac{\omega_{z4} - \omega_0}{\omega_0}\right)^2 - \left(\frac{\omega_{z3} - \omega_0}{\omega_0}\right)^2 - \left(\frac{\omega_{z3} - \omega_0}{\omega_0}\right)^2$$

$$M_{34}^2 = \frac{[(\omega_{z3} - \omega_0)/\omega_0]^2[(\omega_{z4} - \omega_0)/\omega_0]}{M_{12}}$$

$$M_{23}^2 = \left(\frac{\omega_{p3} - \omega_0}{\omega_0}\right)^2 - M_{34}^2.$$

n Cavities

The solution in the general case of arbitrary n may not be unique. However, it is easy to derive a coupling matrix M_n for which the input impedance $Z_{11}^{(n)}$ is specified as in (1). Then certain elements of this matrix (where it is known that no couplings exist) can be reduced to zero using orthogonal similarity transformations, as described in [4]. For this purpose, then, consider the input admittance

$$y_{11}^{(n)}(\lambda) = j \prod_{i=1}^{n-1} (\lambda - \mu_i) \Big/ \prod_{\nu=1}^{n} (\lambda - \lambda_\nu) \qquad (11)$$

where μ_i and λ_ν are the measured (normalized) poles and zeros of $Z_{11}^{(n)}$, respectively; i.e.,

$$\mu_i = \left(\frac{\omega_{pi}}{\omega_0} - \frac{\omega_0}{\omega_{pi}}\right) \simeq \frac{2}{\omega_0}(\omega_{pi} - \omega_0), \qquad i = 1,2,\cdots,n-1$$

$$\lambda_\nu = \left(\frac{\omega_{z\nu}}{\omega_0} - \frac{\omega_0}{\omega_{z\nu}}\right) \simeq \frac{2}{\omega_0}(\omega_{z\nu} - \omega_0), \qquad \nu = 1,2,\cdots,n. \qquad (12)$$

The poles $j\lambda_\nu$ must be simple, purely imaginary, and satisfy the condition [4]

$$\sum_{\nu=1}^{n} \lambda_\nu = 0. \qquad (13)$$

A partial fraction expansion of (11) can now be made as

$$y_{11}^{(n)}(\lambda) = j \sum_{i=1}^{n} \frac{K_i}{\lambda - \lambda_i} \qquad (14)$$

where

$$K_i = \prod_{l=1}^{n-1} (\lambda_i - \mu_l) \Big/ \prod_{\nu=1;\nu\neq i}^{n} (\lambda_i - \lambda_\nu), \qquad i = 1,2,\cdots,n. \qquad (15)$$

Note that all the residues K_i are real, positive, and satisfy the condition [4]

$$\sum_{i=1}^{n} \lambda_i K_i = 0. \qquad (16)$$

The conditions expressed by (13) and (16) are necessary conditions for synchronous tuning of the cavities.

An orthogonal matrix T can now be constructed whose first row is given by

$$T_{1j} = (K_j/N^2)^{1/2}, \qquad j = 1,2,\cdots,n \qquad (17)$$

where

$$N^2 = \sum_{i=1}^{n} K_i$$

is a normalization constant. The remainder of T can be found by a variety of ways; e.g., by using the Gram–Schmidt procedure and the coupling matrix M can be constructed from

$$M = T\Lambda T^t \qquad (18)$$

where $\Lambda = \text{diag}(\lambda_1,\lambda_2,\cdots,\lambda_n)$ and T^t is the transpose of T. The diagonal elements of M as well as other nondiagonal elements can be reduced to zero [4], and hence the required coupling matrix is reconstructed. In particular, if the cavities in the structure being measured are direct coupled (i.e., no cross-couplings exist between nonadjacent cavities), then the reduction of M to a tridiagonal form is always possible and yields the (unique) values of the coupling elements. In practice one can always apply this result in measuring any desired couplings in a multiple-coupled cavity system by the proper choice of a subset of direct-coupled cavities which contains the couplings to be measured, while detuning other cavities to which a cross-coupling path exists.

MEASUREMENT ACCURACY

In practice, since the cavities have finite unloaded Q, a limit will be set on the smallest coupling that can accurately be measured. It is possible to see the qualitative effect of the loss on measurement accuracy by considering two cavities. If the loss is taken into account, the coupling is given by

$$M_{12}^2 = \frac{\Delta\omega_{z12}^2}{\omega_0^2} + \frac{1}{Q^2}.$$

This leads to the approximate rule that M must be at least $10(1/Q)$ to keep the error less than 0.5 percent.

The 0° and 180° phase positions of ρ can be accurately measured by using a network analyzer and by employing a stable CW source together with a digital counter. The final error in M is principally dependent on operator and network analyzer error.

CONCLUSIONS

Fig. 3 is a typical result of the measured coupling between two waveguide cavities. It has been found that the coupling value is not constant, but depends on the resonant frequency of the screw-tuned cavities. The coupling is affected by the penetration of a tuning screw or the presence of additional coupling slots and/or screws. The mode is perturbed, and consequently the coupling is decreased. This result demonstrates very clearly that, for precise realization of a set of coupling slots, it is essential for coupling measurements to be made within the filter while the filter is assembled and while each cavity is tuned to synchronous frequency.

The measurement technique was first used to set the coupling values on the nonminimum-phase optimum-amplitude waveguide bandpass filter [3]. Without such a technique, it would have been

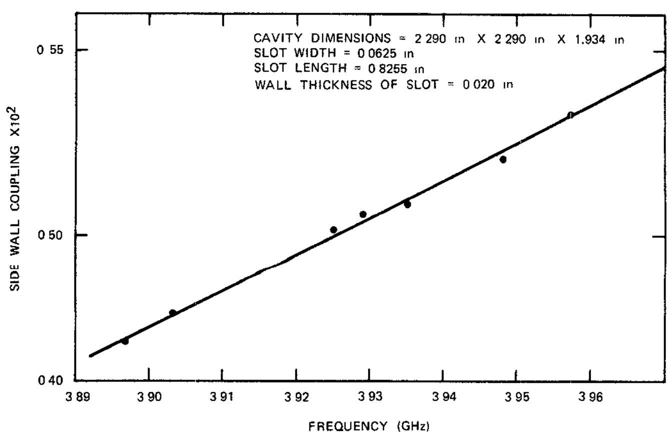

Fig. 3. Side-wall coupling versus cavity synchronous frequency.

impossible to tune the filter due to perturbation of the coupling values by the coupling and tuning screws.

REFERENCES

[1] H. A. Bethe, "Theory of side windows in waveguides," M.I.T. Radiation Lab., Cambridge, Mass., Rep. 43-27, Apr. 1943.
[2] S. B. Cohn, "Microwave coupling by large apertures," *Proc. IRE*, vol. 40, pp. 696–699, June 1952.
[3] A. E. Atia and A. E Williams, "Nonminimum-phase optimum-amplitude bandpass waveguide filters," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-22, pp. 425–431, Apr. 1974.
[4] A. E. Atia, A. E. Williams, and R. W. Newcomb, "Narrow-band multiple-coupled cavity synthesis," *IEEE Trans. Circuits Syst.*, vol. CAS-21, pp. 649–655, Sept. 1974.

Loss Considerations for Microstrip Resonators

E. BELOHOUBEK, FELLOW, IEEE, AND
E. DENLINGER, MEMBER, IEEE

Abstract—**The influence of radiation losses on the Q of microstrip resonators is shown for a variety of frequencies, characteristic impedances, substrate materials, and thicknesses. Radiation becomes a dominant factor at higher frequencies, especially for low-impedance lines and thick substrates with a low dielectric constant.**

I. INTRODUCTION

The designer of microstrip circuits is often faced with the question of what the best substrate thickness is, and what characteristic impedance should be used if low loss is of prime importance. The particular circuit may be a high-Q resonator, a coupled-line filter, or some general matching network. The losses encountered with microstrip circuits divide into conduction, dielectric, and radiation losses. The first two loss factors have been dealt with extensively in the literature and good approximations exist [1]. Radiation losses are less well understood although several articles [2]–[6] have appeared in the last few years, and quantitative solutions are difficult to come by. The most comprehensive theoretical treatment of radiation from various circuit discontinuities was given by Lewin [6] back in 1960, and has been more recently supplemented by Sobol [3], [4].

Lewin's results, applied to an open-ended microstrip line, are used in this short paper to explore the effect radiation has on the overall Q of microstrip resonators for different values of characteristic impedance, frequency, dielectric constant, and substrate thickness. We will also show that by correcting an error in the definition of the fractional radiated power in previous papers [2], [5], the radiation losses based on Lewin's derivation are indeed in reasonably good agreement with experimental results [2] obtained earlier.

II. LOSS CALCULATIONS

Many microstrip circuits include open-circuited matching stubs and $\lambda_g/4$ or $\lambda_g/2$ resonators which have a tendency to radiate substantial amounts of RF power under certain conditions. This radiated power is either lost to the outside in open structures, or may lead to unwanted cross coupling between various circuit elements within a closed housing. Sometimes, lossy damping material is employed to absorb the radiated power and reduce cross coupling in enclosed structures. For the circuit designer, it would thus be very helpful to know in advance what losses he can expect as a function of prime design parameters such as characteristic impendance Z_0, dielectric constant ϵ_r, frequency f, and substrate thickness h.

The various loss contributions for a microstrip resonator can be represented by Q values in the form

$$Q = \frac{2\pi f_0 U}{W} \tag{1}$$

where f_0 is the resonance frequency, U is the stored energy, and W is the average power lost for a $\lambda_g/4$ microstrip resonator as shown in Fig. 1. The overall Q_t of the resonator is given by

$$\frac{1}{Q_t} = \frac{1}{Q_c} + \frac{1}{Q_d} + \frac{1}{Q_r} \tag{2}$$

where

$$Q_c = \frac{2\pi f_0 U}{W_c} = \frac{\pi}{\alpha_c \lambda_g} \quad \text{conductor losses} \tag{3}$$

$$Q_d = \frac{1}{tg\delta}\left(1 + \frac{1-q}{q\epsilon_r}\right) \quad \text{dielectric losses} \tag{4}$$

$$Q_r = \frac{2\pi f_0 U}{W_r} \quad \text{radiation losses} \tag{5}$$

where

- W_c average power lost in the conductors;
- W_r average power lost due to radiation;
- α_c conductor attenuation constant;
- $tg\delta$ dielectric loss tangent;
- q dielectric filling factor (fraction of total fields in the dielectric).

The following assumptions are made in the calculation of the individual Q values with the aim of keeping the results as close as possible to realistic conditions. Thus, dielectric losses, surface roughness, and dispersion are included, although their influence on the overall Q_t in many cases is relatively minor.

1) Dielectric losses: the calculations are based on an alumina ceramic with $\epsilon_r = 10$ and $tg\delta = 10^{-4}$; for comparison, a low dielectric constant material RT Duroid 5870 with $\epsilon_r = 2.35$ and $tg\delta = 10^{-3}$ is also included. The dielectric filling factor q varies typically from 0.5 to 1 which only in the case of low ϵ_r leads to a sizable correction for Q_d, according to (4). For alumina, the effect of q can be neglected.

2) Surface roughness: a uniform surface roughness of 5-μin rms

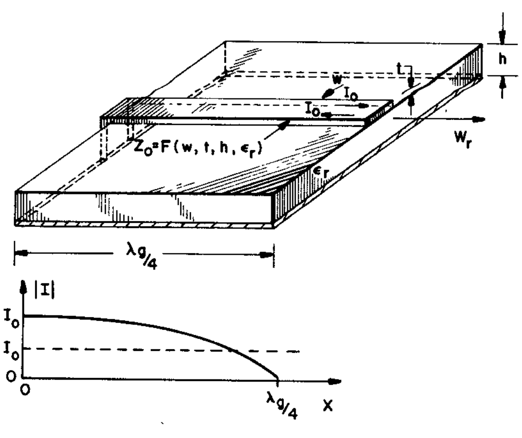

Fig. 1. Quarter-wave microstrip resonator.

Manuscript received July 22, 1974; revised October 24, 1974.

The authors are with the Microwave Technology Center, RCA Laboratories, Princeton, N. J. 08540.